



**Desktop Review of
Mathematics School Education:
Pedagogical Approaches
and Learning Resources**

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Preamble

This paper identifies and discusses key issues influencing student engagement with mathematics. It emphasises the need for integration of curriculum (what mathematics is to be taught and learnt), pedagogy (how that mathematics is to be taught and learnt), and assessment (what mathematics has been learnt, and what that means for future learning).

The focus of the paper provides advice on the nature and targetting of classroom resources that can assist in addressing the drift of students away from mathematics by implementing inquiry pedagogies to provide classroom experiences that are engaging for students.

Teachers having access to high quality resources for the learning of mathematics is essential. But resources are only one part of the picture. At least as important is for teachers to have the commitment and capacity to use those resources. Hence professional learning that complements the availability of high quality resources is also needed. Professional learning should target creating and maintaining a culture in schools that supports and expects teachers to consider and actively explore new ways of teaching and learning in mathematics as part of their everyday professional responsibilities. Teaching resources should be designed¹, where possible, to promote and support professional learning and this culture among teachers of mathematics.

A paper of this length covering such a broad range of issues necessarily describes the general picture. It is acknowledged that there are many classrooms where students currently have exceptional experiences in mathematics – these exceptions prove the rule and can serve to point the way.

The author also acknowledges the support of the many people who have provided input and guidance in the development of the paper.

¹ This paper takes the view that high quality educational resources should be the result of a design process that will commonly involve people with different sets of expertise; and which has cycle(s) of piloting and revision. Hence the terms 'design' and 'designer' are used, rather than 'writing' and 'writer'.

Question 1

Are the current pedagogical approaches to teaching mathematics relevant and/or appropriate in the 21st century knowledge economy, including the extent to which they reflect the contemporary technology-rich environment?

The term ‘knowledge economy’ is much used to describe the world in which we currently live, and for which we are educating our young people. A working definition is that the knowledge economy is one in which “production and services (are) based on knowledge-intensive activities that contribute to an accelerated pace of technical and scientific advance.”² This is vastly different from the industrial society from which it emerged and to which much of current mathematics curriculum, teaching and assessment is most strongly connected. No longer can we expect schooling to provide all – or even most – of what tomorrow’s citizens will need to know and be able to do.

One consequence of this fundamental shift has been the development of pedagogical advice and expectations for teachers from education authorities. These include general frameworks such as the South Australian *Teaching for Effective Learning* (TfEL) framework that “describes the Department of Education and Child Development (DECD) position on pedagogy for all schools”³ and similar documents in other jurisdictions. The Australian Institute for Teaching and School Leadership (AITSL) *Australian Professional Standards for Teachers*⁴ reflect these pedagogical frameworks in their expectations of teachers at different career stages. These and other developments are designed to help ensure that schooling builds young people’s capacity to thrive in the knowledge economy.

Over the last 30 years, mathematics education research has been a burgeoning area in Australia and internationally. A significant amount of that research output constitutes findings about the efficacy of a diverse range of pedagogies for mathematics. Many of these connect directly with the general pedagogical advice and provide mathematics-specific pathways for teachers to meet the requirements. For example, TEfL requires promotion of “dialogue as a means of learning” and that students learn to “communicate learning in multiple modes”. Both these pedagogical orientations are strongly represented in the mathematics education literature.

Contemporary evidence-based advice on mathematics pedagogy is therefore consistent with general requirements for teachers’ pedagogy. However, current approaches to teaching mathematics in our schools are very patchy in terms of implementation of contemporary mathematics pedagogy. For example, using physical materials for the learning of mathematics is well accepted as pedagogically sound, particularly in the primary years. However, teaching approaches that take students from reliance on concrete materials to the more abstract world of mathematics using symbols are essential for student progress through “learning in multiple modes”. Scaffolding this shift is as important as using concrete materials in the first place.

In the secondary years, many schools and teachers use textbooks with pedagogical approaches of varying quality and orientation. When used as the principal (or only) resource for teaching and learning, most current textbooks lead to an overwhelming focus on content knowledge that is learnt through example and practice. Arguably, the 1999 *TIMSS Video Study*, in which lessons from a range of countries were videoed and

² Powell, W. W. & Snellman K. 2004. The knowledge economy. *Annual Review of Sociology* 30, 199–220.

³ <http://www.learningtolearn.sa.edu.au/tfel>

⁴ <http://www.aitsl.edu.au/australian-professional-standards-for-teachers>

analysed⁵, remains reflective of the experience of many current Year 8 students. Analysis revealed a quite depressing picture of the students' classroom experience (see response to Question 4 for an outline of these findings). These sorts of experiences do not connect well with either the general or mathematics-specific advice on pedagogy. On that basis, current approaches that rely heavily on the use of textbooks are not preparing students for effective participation in the knowledge economy.

The definition of the knowledge economy indicates that it goes hand in hand with technologies. Among mathematics education researchers, there is significant interest in the issue of technologies and the learning of mathematics, and clear findings that technologies, when well used in the classroom, can have a positive impact on students' learning. These technologies vary considerably and include young children exploring and learning patterns in our number system through the use of simple four-function calculators; purpose designed online games; other interactive online environments and tools; technologies such as graphics calculators that connect different mathematical representations; using mathematically powerful tools such as spreadsheets, geometry packages, statistical software and computer algebra systems (CAS) that perform multiple and often difficult calculations, thus allowing students to model and work with complex, real-world contexts.

Again, whilst the case for the use of technology as a part of teachers' pedagogy in mathematics is strong, the uptake is patchy. Some Australian teachers are at the leading edge of practice internationally; the majority will generally make some or little use of technologies for teaching mathematics without it being a significant component of their teaching and learning program. Schools will also cite cost as a factor in not being able to make contemporary technologies available for teaching and learning.

The matter of the use of technology for the teaching and learning of mathematics is further complicated by the spectrum of uses. For example, the Khan Academy⁶ "offers practice exercises (and) instructional videos" that replicate traditional classroom practice in an online form that is available "anywhere...anytime". In contrast, *Computer Based Math*⁷ uses the premise that "(r)real-life math has been transformed by computer-based calculation; now mainstream math education needs this fundamental change too" and builds curriculum and classroom delivery accordingly, thereby enabling students to engage with and explore "hard questions across many contexts". These both exploit current and emerging technologies, but in fundamentally different ways and for seemingly diametrically opposed purposes.

Hence the picture is one of a mismatch between the contemporary advice on mathematics pedagogy on the one hand, and current practices on the other. This is true in general as well as in the specific case of the use of technologies. There are many reasons for this. Some of them, such as teachers' awareness of alternatives, a sense of lack of support to 'have a go' with something new, having further examples that build confidence and capacity with a new approach are able to be addressed through carefully designed resources that have professional learning orientation. Other reasons for the mismatch, such as teachers needing time and collegial support to develop their practice in mathematics and assessment practices at the senior secondary level, go well beyond the scope of a project to develop classroom resources.

⁵ Hollingsworth, H., Lokan, J. & McCrae, B. (2003). *Teaching Mathematics in Australia. Results from the TIMSS 1999 Video Study*. Camberwell, Vic.: ACER.

⁶ <https://www.khanacademy.org/about>

⁷ <http://computerbasedmath.org>

Question 2

What is the role of inquiry-based pedagogy in the teaching of mathematics (including across the age groups and across mathematical disciplines)?

Inquiry-based pedagogy is the central organising theme for science in Australian schools. The Rationale for the *Australian Curriculum: Science* (ACS)⁸ defines science as:

...a dynamic, collaborative and creative human endeavour arising from our desire to make sense of our world through exploring the unknown, investigating universal mysteries, making predictions and solving problems.

and this leads, in the section of the ACS on implications for teaching, assessment and reporting⁹, to an unequivocal commitment to inquiry-based pedagogy:

The science curriculum emphasises inquiry-based teaching and learning. A balanced and engaging approach to teaching will typically involve context, exploration, explanation and application. This requires a context or point of relevance through which students can make sense of the ideas they are learning. Opportunities for student-led open inquiry should also be provided within each phase of schooling.

In mathematics, the term ‘inquiry’ came to prominence when the *National Statement on Mathematics for Australian Schools* (1990)¹⁰ identified Mathematical Inquiry as one of the seven strands in the first attempt at a national curriculum. Whilst the term has been used somewhat in Australia¹¹ it is not widespread. However, there are clear parallels between science inquiry-based approaches and contemporary thinking about pedagogy in mathematics, whether it is described as ‘inquiry-based’ or not.

In his 2011 ACER Monograph, *Teaching Mathematics: Using research informed strategies*¹², Peter Sullivan identifies “six (key) principles for (effective) teaching of mathematics which are specific to mathematics, but which are also based on sound general pedagogic principles” and which are “re-enforced by much of the research”(p. 24).

Sullivan’s six principles are a well-respected¹³ summary statement of pedagogical advice. They are:

- Principle 1: Articulating goals
- Principle 2: Making connections
- Principle 3: Fostering engagement
- Principle 4: Differentiating challenges
- Principle 5: Structuring lessons
- Principle 6: Promoting fluency and transfer

⁸ <http://www.aitsl.edu.au/australian-professional-standards-for-teachers>

⁹ <http://www.australiancurriculum.edu.au/science/implications-for-teaching-assessment-and-reporting>

¹⁰ Australian Education Council. (1990). *National statement on mathematics for Australian schools*. Carlton, Vic.: Curriculum Corporation.

¹¹ For example “implementing mathematical inquiry, while highly promising as a pedagogical practice” in Makar, K. (2011). Learning over time: Pedagogical change in teaching mathematical inquiry. In J. Clark, B. Kissane, J. Mousley, T. Spencer & S. Thornton (Eds), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 349–359). Adelaide, SA: AAMT & MERGA.

¹² Sullivan, P. (2011). *Teaching Mathematics: Using research informed strategies*. Australian Education Review. Camberwell, Vic.: ACER Press.

¹³ It is noteworthy that Sullivan was the lead writer of the ACM.

Inquiry-based pedagogy in mathematics is a practical vehicle for implementing these principles. The first four principles are clearly central to the philosophy and practice of inquiry-based pedagogy in mathematics and will be evident in resources designed to exemplify, articulate and promote this approach to teachers. Principle 5 will need to be reflected as practical support for teachers through classroom materials promoting inquiry-based pedagogy. Principle 6 can, and should be, part of the design of the materials to ensure fidelity with Sullivan's overall framework, even though perhaps not strictly necessary to the concept of inquiry-based pedagogy.

Currently the *Australian Curriculum: Mathematics* (ACM) has four strands of Proficiencies which many believe are the 'core' purpose of mathematics – Reasoning, Problem solving, Understanding and Fluency. Development in the first three of these is central to well-designed inquiry approaches, and Sullivan's Principle 6 highlights the importance of incorporating development of Fluency in the approach. Equally, the inquiry approach is applicable to all three Content strands of the ACM – Number and algebra, Geometry and measurement, and Statistics and probability. As a proof of concept for this, online resources such as *Maths300*¹⁴ and *NRICH*¹⁵ contain examples of inquiry approaches in diverse areas of mathematics.

Maths300 and *NRICH* also illustrate that inquiry-based pedagogy is applicable and achievable across the year levels. That said, the discussion later in this paper will identify year levels and other areas in which there are particular gaps. It is also important to note that, whilst inquiry-based pedagogy in mathematics has much to recommend it, students benefit from learning the subject through a range of pedagogies. There are other approaches that have been shown to be effective (see response to Question 5 where some of these are identified). An emphasis on inquiry approaches should not be portrayed as devaluing these other approaches in the eyes of teachers.

The matter of students' destinations can come into play in the junior secondary years. For those on a pathway to higher-level mathematics in their senior years, teachers often claim that there is no time for other than transmission pedagogies given the amount of content to be covered. However, the benefits of inquiry-based approaches are important for students' mathematical futures, particularly in terms of the development of mathematical skills and orientations to problem solving. To limit these students' exposure to pedagogies that promote 21st century skills such as problem solving, reasoning, and creative and critical thinking is counter-productive.

¹⁴ <http://www.maths300.esa.edu.au>

¹⁵ <http://nrich.maths.org/frontpage>

Question 3

Do current approaches to assessment in mathematics need to be revised to better reflect problem-solving pedagogical approaches?

The answer to this question is a simple and unequivocal “Yes”. One needs to look no further than calls for a broadening of assessment strategies by influential leaders in education, both internationally (Barber and Hill, 2015)¹⁶ and here in Australia (Masters, 2013)¹⁷, to conclude that current approaches and emphases are seen to be antipathetic to supporting development of 21st century skills. This view is also reflected in education authorities’ advice on assessment.

The need for coherence between curriculum, pedagogy and assessment has been discussed earlier in this paper. Current approaches to assessment in mathematics are arguably even more disconnected from current advice on pedagogical approaches than is the norm in other areas of the curriculum. For example, the current emphasis on testing (outlined below) means that assessment of Problem solving (one of the four Proficiencies mandated in the ACM) is generally limited to situations involving one- and two-step problems that can be incorporated into a test. Even if teachers are using rich contexts or open-ended questions to promote complex problem solving in mathematics, their assessment of the students’ problem solving skills in these contexts tends to be both limited and undervalued. This can have a negative impact on the curriculum implemented in the classroom because “(t)eachers struggle to follow through with high quality pedagogical interactions... whilst also adhering to the quality control measures of testing” (Dimarco, 2009)¹⁸.

There are at least two reasons for the continuing mismatch between curriculum and assessment. The first is historical: the ‘maths test’, with its emphasis on skills and procedures, has been the pillar of assessment in mathematics for generations of students, particularly in secondary schools. The second reason is more recent and relates to primary schooling. It is the advent of the *National Assessment Program – Literacy and Numeracy* (NAPLAN) numeracy testing. The importance attached to results and data from NAPLAN has reinforced the view that assessment is testing. Many primary schools now have a more test-oriented headset since the advent of NAPLAN.

This is not an argument against testing. Tests are efficient means for obtaining information about some aspects of a child’s learning and should be used for these purposes. The leading authors above argue that assessment should be a contributor to learning by informing teachers’ decision-making about their students’ future learning. This requires using approaches that are sensitive to other aspects of students’ learning, aspects that are not amenable to measurement through the testing genre. Many 21st century skills – problem solving among them – are not often evident when students have to try to work out a solution in isolation, in silence and against the clock (i.e., in a traditional test).

In the 1980s, when the development of electronic word processors was focussed on achieving What You See Is What You Get (WYSIWYG), Hugh Burkhardt from the UK

¹⁶ Hill, P. & Barber, M. (2014). *Preparing for a renaissance in assessment*. Pearson: New York.

¹⁷ Masters, G. (2013). Reforming educational assessment: Imperatives, principles and challenges. *Australian Education Review*. Camberwell, Vic.: ACER Press. Downloaded from <http://research.acer.edu.au/cgi/viewcontent.cgi?article=1021&context=aer> on 16 March 2015.

¹⁸ Dimarco, S. (2009). Crossing the divide between teacher professionalism and national testing in middle school mathematics? In R. Hunter, B. Bicknell, & T. Burgess (Eds), *Crossing divides* (Proceedings of the 32nd annual conference of Mathematics Education Group of Australasia, pp. 451–458). Palmerston North, NZ: MERGA.

coined the term *WYTIWYG* (What You Test Is What You Get)¹⁹ to make the point that, in mathematics, assessment tends to drive both what is taught and how it is taught. This realisation led to experiments with examination boards in England to include a small number of challenging problem-solving items, and back these by providing innovative classroom curriculum materials and extensive teacher support. Since students' problem solving was to be tested in the examinations, it was taught using the curriculum materials. These units remain highly regarded and awarded examples of problem solving approaches known as the Shell Centre 'Blue Box'²⁰ and the 'Red Book'²¹.

Hence, not only should assessment be well-connected to curriculum and pedagogy, the assessment 'tail' can wag the curriculum and pedagogy 'dog', and do so in positive ways.

¹⁹ Burkhardt, H. (1987). Curricula for active mathematics. In I. Wirszup, & R. Streit (Eds), *Developments in school mathematics around the world 1* (pp. 321–361). Reston, VA: National Council of Teachers of Mathematics.

²⁰ Swan, M. (1984). *Problems with patterns and numbers*. Nottingham, UK: The Shell Centre for Mathematical Education.

²¹ Swan, M. (1985). *The language of functions and graphs*. Nottingham, UK: The Shell Centre for Mathematical Education.

Question 4

What can we learn from overseas about the teaching of mathematics that is relevant to education in Australia, particularly from those countries which are performing better than Australia in TIMSS and PISA?

There are clear signs that there is room for improvement in students' learning of mathematics. Internally, there is the continuing focus on skills shortages in the STEM area in the context of increasingly technology-rich contemporary workplaces (i.e., the knowledge economy), as voiced by industry, for example by the Australian Industry Group²². Externally, Australian students' performance in the Program for International Students Assessment (PISA) is evidence of this. Given that PISA assesses "to what extent students at the end of compulsory education, can apply their knowledge to real-life situations and be equipped for full participation in society"²³, it is clear that PISA is measuring some components of readiness for the knowledge economy of the 21st century. Hence PISA results are important data to consider and respond to.

The cause of Australian students not performing as well as their counterparts from other countries could be related to the curriculum being used. Alternatively, it could be the result of the teaching practices being used – or a combination of the two.

Turning first to the curriculum. During the development of the ACM, the Australian Curriculum, Assessment and Reporting Authority (ACARA) commissioned a project to internationally benchmark the curriculum against several others across the world. The findings of that analysis were that the Australian curriculum was comparable to those of the other countries²⁴.

More recently, the 2014 *Review of the Australian Curriculum* undertook a more detailed, qualitative review of the ACM against the curriculums in Japan, Singapore and USA²⁵. The most telling finding of that review is that the ACM is uneven in its focus on key ideas, suggesting that "the priority given to some key ideas and skills could be sharper" (Stephens, p. 55). Among the key ideas from the primary years discussed by Stephens, two stand out as being fundamental to students' progress in mathematics and numeracy. These are multiplicative reasoning and algebraic foundations in number.

Good grounding in both of these are essential for students to progress to using algebraic symbols, processes and conventions effectively and with understanding. Multiplicative reasoning underpins algebraic reasoning, as it does for many aspects of the functional numeracy required for participation in society and work. Algebraic reasoning is important in junior secondary mathematics, and crucial in the higher levels of mathematics in the senior years and beyond.

Whilst it is beyond the scope of this paper (or the project it foreshadows) to expect changes to the current ACM, the resources developed by the *Mathematics by Inquiry* project can be designed to interpret the ACM in ways that give an emphasis to the 'big ideas'²⁶ of mathematics through successive years of schooling.

²² Australian Industry Group. (2013). *Lifting our Science, Technology, Engineering and Maths (STEM) Skills*. Melbourne, Vic.: Author.

²³ *What makes PISA different*. Downloaded 2 March 2015 from <http://www.oecd.org/pisa/aboutpisa>

²⁴ Personal briefing from senior ACARA personnel on unpublished report.

²⁵ Australian Government. (2014). *Review of the Australian Curriculum: Supplementary material*. Downloaded 27 February 2015 from <http://docs.education.gov.au/node/36271>

²⁶ The term 'big ideas' is used throughout this paper to refer to major conceptual understandings in mathematics that are necessary for students to progress in their learning. There are differing views among mathematics educators about the details of what constitutes a 'big idea', the notion that there are key concepts in mathematics that underpin progress is widely accepted.

The *TIMSS Video Study* mentioned previously, maps out some clear differences in teaching practices between the countries studied. The 100 Australian mathematics lessons in the study at year 8 level were characterised by a “shallow teaching syndrome: procedures without reasons” that involved “excessive repetition”, “problems of low complexity” and “absence of mathematical reasoning” (pp. 119–120). These were in marked contrast to the classrooms in Japan and Hong Kong where reasoning was much more in evidence. As an aside, the resources used in 91% of the Australian classrooms were textbooks or worksheets (p. xvi). Hence there is an issue about the quality of teaching of mathematics to which well-designed resources can contribute.

In his address at the 2012 conference of the Mathematics Education Research Group of Australasia (MERGA)²⁷, Frederick Leung, a distinguished mathematics educator from Hong Kong, sounded a cautionary note about simplistic adoption of strategies and practices from those East Asian countries (including Hong Kong) that currently dominate the upper echelons of performance in PISA. He noted that these countries share a Confucian tradition with a particular approach to, and reverence for, education. Much of the practice of mathematics education in those countries depends on those traditions and is unlikely to be directly transferable to other countries. He advised those in other countries to consider the teaching of mathematics in high performing countries with a view to modifying and adapting what we learn into our own context and traditions.

Finland’s 15 year olds have also consistently performed extremely well in PISA mathematics assessments. In general this success is seen to be most strongly influenced by “excellent teachers and high quality teacher education”²⁸ in a context in which teaching is a highly valued profession. More broadly, the McKinsey report *How the world’s best-performing school systems come out on top*²⁹ confirms the focus on teachers and teaching as key influences on student performance in its “three things (that) matter most:

- getting the right people to become teachers;
- developing them into effective instructors; and
- ensuring that the system is able to deliver the best possible instruction for every child.” (p. 2)

Shifting teaching practices towards inquiry-based approaches can be supported by appropriately designed resources. Targetted and sustained professional learning of teachers is required to cement this change, however.

²⁷ Leung, F. (2012). What can and should we learn from international studies of mathematics achievement? In J. Dindyal, P. Lu & F. Swee (Eds). *Mathematics education: Expanding horizons* (Proceedings of the 35th Annual Conference of the Mathematics Education Research Group of Australasia). Adelaide, SA: MERGA.

²⁸ Simola, H. (2005). The Finnish miracle of PISA: Historical and sociological remarks on teaching and teacher education. *Comparative Education* 41(4), 455–470.

²⁹ McKinsey Company. (2007). *How the world’s best-performing school systems come out on top*. Downloaded 25 March 2015 from <http://mckinseysociety.com/how-the-worlds-best-performing-schools-come-out-on-top>

Question 5

Which pedagogical approaches have been shown to work with specific groups under-represented in advanced mathematics at senior secondary level (girls, Indigenous, disadvantaged students)?

In a study of senior secondary students' views on the teaching of mathematics, Helme and Teese³⁰ found that they express preferences for “teaching methods be less dependent on textbooks, more interactive, and with a slower pace of teaching... more enjoyable coursework, course content with stronger links to real life situations, and more *hands on* approaches to learning” (emphasis in original). These findings provide clear pointers to general features of pedagogical approaches that appeal to students.

Collective Argumentation (CA) is an example of a pedagogical approach that has demonstrated some success in the senior secondary years. Collective Argumentation is described as students making and critiquing arguments within their mathematics classes. This learning often occurs during class discussions, in which arguments are made public for all students in the class.

In an Australian study, not only were students in senior mathematics classes more engaged in the mathematical learning when CA was incorporated, but also their sense of agency was enhanced. Enacting CA supported students' increased participation in the task and with each other, as well as in important mathematical practices.³¹ CA is one way of generating meaningful mathematical discourse in the mathematics classroom – the general inference is that approaches which support and expect discussion between students can engage students by presenting learning as a social and collaborative enterprise. This has long been recognised as important in retaining girls, in particular, as active participants in mathematics. It is reinforced by findings about Australian students and science reported in response to Question 7.

Using real contexts, and connecting learning to students' lives and interests, is likely to support Indigenous students becoming more successful mathematics learners.³² For these students the connection to their lives includes their community, culture and, often, 'country'. In the literature on Indigenous students and learning mathematics, other practices supported include cross-age tutoring, engaging elders as the respected knowledge keepers in the community, and group work when it supports deep learning by being “structured so as to enable learners to talk, debate, contest, clarify etc. their understandings as they engaged with mathematical tasks that were cognitively demanding”³³. These comments about the nature of discourse are noticeably similar to those of CA. The closing comment that tasks should be cognitively demanding, reflects a common recommendation that educators should set high standards for Indigenous learning in mathematics. Whilst this is relevant to all students, researchers presumably feel a need to reiterate the sentiment because at least some teachers apparently do not do so.

³⁰ Helme, S. & Teese, R. (2011). How inclusive is year 12 mathematics? In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds), *Mathematics: Traditions and [new] practices* (proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd Biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 349–359). Adelaide, SA: AAMT & MERGA.

³¹ Redmond, T. & Sheehy, J. (2009). Reconceptualising agency in senior mathematics classrooms. In R. Hunter, B. Bicknell, & T. Burgess (Eds), *Crossing divides* (Proceedings of the 32nd annual conference of Mathematics Education Group of Australasia, pp. 451–458). Palmerston North, NZ: MERGA.

³² Warren, E., Cooper, T. J. & Baturu, A. (2009). Bridging the Educational gap: Indigenous and non-Indigenous beliefs, attitudes and practices in a remote Australian school. In J. Zajda & K. Freeman (Eds), *Race, ethnicity and gender in education* (pp. 213–226). New York, NY: Springer.

³³ Jorgensen, R. (2009). The use of home language in the mathematics classroom. In R. Hunter, B. Bicknell, & T. Burgess (Eds), *Crossing divides* (Proceedings of the 32nd annual conference of Mathematics Education Group of Australasia, pp. 451–458). Palmerston North, NZ: MERGA.

The concept of ‘Culturally Responsive Mathematics Pedagogy’ (CRMP) has emerged in relation to Indigenous learners in recent years. AAMT’s *Make it Count* project used this as the frame for its research and development of practices in eight clusters of metropolitan and regional schools. The project’s findings³⁴ are mapped against the Australian Professional Standards for Teachers (AITSL)³⁵ and articulate and exemplify aspects of CRMP as they were identified in the diverse schools involved.

The *Make it Count* findings about CRMP are extensive and address the three domains of the standards (professional knowledge, practice and engagement). The findings that reflect the pedagogical orientations the project describes as CRMP include:

- mathematics learning that is intentionally culturally relevant/connected as well as academically rigorous;
- mathematical concepts that build on, connect with, and lead to other concepts;
- linking mathematics to contexts beyond the classroom so it can be taught through rich, life-like activities;
- creating learning experiences that have family and community significance;
- scaffolded teaching with a defined and planned learning goal;
- explicitly teach learners mathematical language and symbols;
- build on student voice, recognition of diverse ways of learning mathematics and connection to worlds beyond the classroom (both local and non-local);
- kinaesthetic pedagogical practices as a point of entry to abstract mathematics knowledge;
- using narrative and discussion to allow students to feel personally connected to the mathematics;
- students representing their learning and thinking in a range of formats (e.g., verbally, physically, symbolically and with technologies);
- encouraging risk taking;
- older students as mentors for their younger peers.

Whilst these pedagogical approaches emerged through work specifically designed to improve the teaching and learning of Indigenous students – and some seem quite specific in their reference to culture for example – many educators have responded to these findings suggesting that it is a description of good teaching of mathematics for any group of students³⁶. This view is broadly supported across the mathematics education literature. Hence, whilst they are derived from work with Indigenous students and therefore directly informative of effective pedagogies for that group of students, the pedagogies of CRMP are a succinct summary of a number of pedagogies that are generally inclusive.

In response to the findings above, and others such as Hunter (2008), resources can support teachers’ work with diverse learners. In work that resonates with aspects of the *Make it Count* findings, Hunter noted the “importance of teachers scaffolding a diverse range of students to use specific questions and prompts promoted their engagement in a range of important mathematical practices” (pp. 197–198). Such questions and prompts can be embedded in the resources in order to address diverse backgrounds and learning needs in practical ways in the classroom.

³⁴ Australian Association of Mathematics Teachers (AAMT). (2013). *Cluster Findings Make it Count: Numeracy, mathematics and Indigenous learners*. Downloaded 25 March 2015 from <http://mic.aamt.edu.au/Findings>

³⁵ <http://www.aitsl.edu.au/australian-professional-standards-for-teachers>

³⁶ The author’s personal observations of responses to the project. The author was Director of the *Make it Count* project.

Question 6

How well is the teaching of mathematics across primary and secondary school supported by existing resources linked to the Australian Curriculum?

Reports from the field and observations of schools indicate that there are ample resources for F–10 (free, and at a cost; hard copy and online) that are linked to the *Australian Curriculum: Mathematics*³⁷. In fact, a common complaint from teachers and schools is that there are too many resources to choose from. The notion of curation of resources against agreed criteria for quality and applicability often arises as a means to help teachers sort the ‘wheat from the chaff’, with there being a sense among many that there is a great deal of ‘chaff’ available. However, few resources are universally applicable and valued. Variation in teaching contexts, students and teachers abound, with the consequence that local factors must inform resource selection for the classroom to some extent.

There are three principal reasons for the plethora of resources available for mathematics.

A study by ACARA (then the National Curriculum Board) to inform the development of the ACM found that curriculums previously in place in the jurisdictions had around 90% of their content in common³⁸. Hence, resources previously developed for any one jurisdiction have only needed relatively minor adjustment to align with the ACM, thereby achieving national reach.

The second reason relates to the increasing number of resources delivered online. Commonality of mathematics content is not only a feature within Australia; internationally there is a great deal of overlap in mathematics curriculums as well. Overseas developers are finding that it is a relatively easy step to ‘tweak’ their resources in electronic form – for example by converting to metric units and connecting the resource to the ACM – with the reward that a whole new national market is opened up. Specialising materials for Australia in this way is prohibitively expensive for hard copy materials, given our population.

The third reason is that mathematics is a universal area of the curriculum that attracts sustained interest. Developers continue to work on and release new resources as new ideas emerge and/or commercial opportunities arise.

However, there is a real question about the extent to which the resources capture the flavour and intent of the ACM in the context of the overall intent of the Australian Curriculum, linked as it is to the goals of the Melbourne Declaration. Whilst it is true that many, perhaps most, resources are linked to the ACM, that linking is invariably to the Content Descriptions, the content of the curriculum. Much less attention is paid to the other dimensions of the ACM – the Proficiencies, General Capabilities and Cross-curriculum Priorities. Some of the vast array of resources deal with these dimensions well. The majority, particularly textbooks and worksheets, are likely to pay little attention to these other components as their purpose is the development of skills and procedures.

³⁷ The uneven uptake of the ACM in the senior years across the country means that there are more limited numbers of resources that are currently expressly linked to the curriculum. This is likely to change as the jurisdictions move to implement the ACM in the senior years over the next few years.

³⁸ Personal briefing from senior ACARA personnel on unpublished report.

Question 7

To what extent do existing resources engage the full range of students (girls, Indigenous, different learning styles)?

‘Engagement’ has become something of a buzz-word in education in general, and in mathematics in particular. In some circles it may be seen as an end in itself, but the only point of seeking to have students engaged is for them to learn. The NSW Department of Education and Training (DET; now Department of Education and Communities) captured this thinking in its description of engagement “as a deeper student relationship with classroom work” (as quoted in Attard 2011)³⁹. The use of the term relationship is not accidental – relationships have been found to be a significant factor in students’ engagement in mathematics in the middle years, the time when dis-engagement becomes prevalent, and especially for girls (Attard, op. cit.).

Considering engagement of students often emerges when its opposite, dis-engagement, becomes apparent. The *Maths? Why not?* project⁴⁰ (McPhan et al., 2008) studied the reasons for students not choosing higher level mathematics in their senior years at school. The level of a student’s engagement in mathematics was identified as a key issue in their decision-making about future study and work options. The study found “engagement (is) an outgrowth of previous experiences. There is an implication that students choose higher-level mathematics during secondary school, and as a post-secondary option, if experiences at lower levels are appropriate” (p. 49). This suggests that attention to student engagement with mathematics needs to occur in the junior secondary and primary years.

‘Fostering engagement’ is the third principle for effective teaching of mathematics elaborated by Sullivan⁴¹. He describes it as “fundamentally about seeking to make mathematics learning interesting for students” (p. 26).

Drawing from a number of sources he paints a picture of engagement as being linked to:

- (b)uild(ing) on what students know, mathematically and experientially, including creating and connecting students with stories that both contextualise and establish a rationale for the learning;
- utilising a variety of rich and challenging tasks that allow students time and opportunities to make decisions, and which use a variety of forms of representation;
- more exposure to less repetitive, higher-level problems, more discussion of alternative solutions, and more opportunity to explain their thinking;
- opportunities to appreciate connections between mathematical ideas and to understand the mathematics behind the problems they are working on;
- appropriate challenges and challenging learning through questioning and higher order questions;
- using a range of practical contexts and representations having high expectations; and

³⁹ Attard, C. (2011). The influence of teachers on student engagement with mathematics during the middle years. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds), *Mathematics: Traditions and [new] practices* (Proceedings of the 34 annual conference of the mathematics Education research Group of Australasia and the 23rd Biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 349–359). Adelaide, SA: AAMT & MERGA.

⁴⁰ McPhan, G., Morony, W., Pegg, J., Cooksey, R. & Lynch, T. (2008). *Maths? Why Not?* Final Report prepared for the Department of Education, Employment and Workplace Relations (DEEWR). Armidale, NSW: SiMERR

⁴¹ Sullivan, P. (2011). Teaching mathematics: Using research informed strategies. *Australian Education Review*. Camberwell, Vic.: ACER Press.

- using ‘worthwhile tasks’ which is interpreted to mean they are meaningful and relevant to the students.

There are clear connections between these factors for encouraging engagement and the pedagogical orientations for culturally responsive mathematics pedagogy (see response to Question 5).

Another key to engagement is student self-efficacy. “Mathematics self-efficacy and valuing were consistent predictors of mathematics engagement shifts, with higher self-efficacy and valuing associated with increases in engagement” (Anderson et al., 2015, p. 228)⁴².

One goal when designing resources must be to engage students in learning. Students who lack understanding, and have a negative personal sense of self-efficacy that comes through lack of success in learning, necessarily find it difficult to be truly engaged. They may do an activity because it is enjoyable, and this is positive, but, in comparison with engagement and success with the learning the activity is designed to enable, enjoyment only is hollow and unsustainable. The level of student engagement that is generated by a particular resource will vary among students, often dramatically. Some of this variability will be connected to general student disengagement with mathematics.

Context is seen as an important element in engagement. For rural students, contexts that are local are important; for Indigenous students, contexts that provide connections with the students’ culture, ‘country’, communities and families are similarly useful and engaging. Resources that incorporate these contexts can be geographically limited. Further, there is the social dimension of contexts as identified by Mike Askew in his Foreword to the Sullivan paper mentioned previously. He notes that “a ‘social perspective’ is more than simply the application of previously learnt mathematics to ‘realistic’ contexts; it also generates the potential that using students’ familiarity with the social context can lead to engagement with the mathematics” (p. iv).

Hence, resources that illustrate the development and use of local contexts (physical and social) would be the only practical approach in a national project to prepare resources for teaching mathematics. Such resources would have a strong professional learning orientation and purpose.

An analysis⁴³ of “the relationship between four inquiry-based teaching practices (use of: (1) models or applications, (2) hands-on activities, (3) interaction and (4) independent investigations) and science achievement” in the PISA 2006 study may give some pedagogical pointers for mathematics. In Australia, the practices of using models, and interaction, were positively correlated with science achievement, while the use of hands-on activities and investigations showed negative correlations with achievement. Interestingly, the trends for Australian students with interaction and hands-on activities were the reverse of those in most of the other seven countries in the study. Further, when the influences of these factors on the achievement of girls and boys are compared, girls’ science achievement is more strongly positively correlated with increased use of applications than that of boys, and more negatively correlated with increased use of investigations than that of boys. These findings support the use of contexts – which necessarily require application of mathematics – to encourage greater engagement and achievement of girls. For both boys and girls a focus on interaction seems warranted.

⁴² Martin, A., Way, J., Bobis, J. & Anderson, J. (2015). Exploring the ups and downs of mathematics engagement in the middle years of school. *Journal of Early Adolescence* 35(2), 199–244.

⁴³ Gee, K. & Wong, K. (2012). A cross national examination of inquiry and its relationship to student performance in science: Evidence from the Program for International Student Assessment (PISA) 2006. *International Journal of Educational Research* 53, 303–318.

Question 8

Where might specific targetting of resources be most helpful? (Consider age groups, numeracy skill acquisition)

There are number of factors that bring the development of new resources for teaching mathematics onto the agenda. These include:

- different emphases in the curriculum;
- developments in pedagogy based on new research findings;
- advances in technology, either in the capability of mathematical software or to the devices used in teaching and learning; and
- finding new and better ways to implement established effective pedagogies.

All these factors are currently in place in school mathematics in Australia; developing resources to respond to them is inevitable. For governments, developing resources targetted at needs is a logical program response that can achieve the current policy imperative of improving the quality of mathematics teaching.

The following assumes that resources will be targetted at being helpful *to teachers*. This seems obvious, but resources will only be helpful if teachers are aware of them, and actually use them with their students in ways that are consistent with the intentions of the resources. These conditions – awareness of the resources, willingness to use them in the classroom and using them in ways that are consistent with their design – rely on the professional engagement of the teacher. That is, resources will only be helpful to most teachers if they are embedded in a professional culture in which teachers are committed to, and supported in, their professional growth.

Age groups

The recognition of the importance of early numeracy (and literacy) learning as underpinning successful progress through schooling, was reinforced with the establishment of a literacy and numeracy agenda by the Australian Government in the mid-1990s. There has been significant research and development effort in the early years since that time. In comparison with some other age groups, the early years (say Foundation to Years 2 or 3) is relatively well served by resources, particularly in number, the core area of mathematics in which young children need a strong foundation for further learning.

The *Developing Efficient Numeracy Strategies* (DENS) Stage 1 and 2⁴⁴ are good quality resources that were result of this effort in the early years. DENS 1 and 2 were developed by the NSW Department of Education and Communities in the late 1990s and early 2000s to translate an extensive research base⁴⁵ into classroom activities and useable sequences of learning for teachers. These books remain highly valued and extensively used across the country.

Similarly, there has been significant development of resources to support the teaching of mathematics further into the primary years. Whether these have involved education authorities in extensive research and development initiatives (such as WA's

⁴⁴ NSW Department of Education and Training. (2014). *Developing efficient numeracy strategies: Stage 1* (2nd edition). Adelaide, SA: AAMT.,
NSW Department of Education and Training. (2004). *Developing efficient numeracy strategies: Stage 2*. Sydney, NSW: Author.

⁴⁵ The *Count Me In* and *Count Me In Too* projects conducted by the Department from the late 1990s. These projects drew extensively on other research and involved a number of significant researchers in the area working with teachers and schools.

*First Steps in Mathematics*⁴⁶) or innovative commercial products that provide teachers with resources to engage and motivate their students' learning⁴⁷, there is a solid bank of well-designed resources for the primary years. The curation issue mentioned earlier certainly comes into play for time-poor teachers, given the large numbers of activities available in print and online.

Turning to the junior secondary years, commercial effort has generally been focussed on textbooks, as these are the predominant genre of resource used at this level. Over the past two decades or so, government-funded research and development has concentrated mainly on F–6 with much less emphasis on junior secondary years. Of course, there are exceptions with this work often being labelled as 'middle years'.

Hence, targetting resources for the junior secondary years 7–10 that are designed as richer alternatives to textbook approaches would seem to be most useful.

Nature of resources

As has been argued earlier, resources available to teachers of mathematics should encompass an emphasis across all aspects of the ACM: content, Proficiencies, General Capabilities and Cross-curriculum Priorities. Given that attention to some of the proficiencies (Reasoning, Problem solving, Understanding), as well as many aspects of the General Capabilities, is wanting in the current suite of resources, targetting of resources to these areas, particularly in the junior secondary years, will be helpful.

The discussion in response to Question 4 about the ACM lacking consistent explicit emphasis on the 'big ideas' in comparison with curriculums from other countries suggests another resource orientation. One way to address this apparent shortcoming of the ACM is to design resources that enable teachers to address relevant Content Descriptions in ways that, through the teaching and the students' experiences and learning, provide a clear articulation of, and focus on, the 'big ideas', including:

- multiplicative thinking and reasoning;
- arithmetic underpinnings and transition to abstract algebraic thinking;
- variability in data;
- mathematical modelling; and
- transformation, patterns and relationships, symmetry (as recurring themes across aspects of mathematics).

Resources that support vocationally-oriented students to transfer and apply their mathematical skills in different contexts would be a useful response to the finding⁴⁸ that this is a critical factor that limits the success of many young people as they move from school to a variety of contemporary workplaces. The report of the *Quantitative Skills in 21st Century Workplaces* project also highlighted some other specific areas of need for vocationally-oriented students in Years 10–12. These are the flexible and reasonably high-level use of spreadsheets in contemporary workplaces, and the notion of 'executive functions' as important work skills that should be developed in the context of mathematics.

⁴⁶ <http://www.det.wa.edu.au/stepsresources/detcms/navigation/first-steps-mathematics/?oid=MultiPartArticle-id-13603817>

⁴⁷ For example, Dr Paul Swan from WA is a prolific author whose many books of classroom activities often use practical materials (cards, dice, dominoes) to stimulate interest, enjoyment and learning through problem solving in the primary years.

⁴⁸ Most recently highlighted in the report of *Quantitative Skills in 21st Century Workplaces* project, downloaded 12 March 2015 from http://www.aamt.edu.au/content/download/31677/446931/file/Quant%20Math%20Skills_Final%20Report.pdf

Question 9

How well do existing resources support out-of-field teachers in developing their mathematics teaching skills?

Support for out-of-field teachers of mathematics is mostly a matter for their professional learning. Carefully designed resources can play a role, however.

The term 'out-of-field' teachers is applicable to years 7–12, and particularly the junior secondary years. Hence, discussion of resources that can support development of these teachers' mathematics teaching skills is, by definition, located in the context of resources for high school teaching of mathematics.

Concern about out-of-field teachers is driven by the finding that up to 40% of students⁴⁹ in years 7–10 are being taught by people without suitable preparation to teach mathematics. It is argued that because of their lack of background in the subject, many of these out-of-field teachers are unable to teach mathematics well, in ways that inspire, encourage and enable students to continue with their study of higher levels of mathematics⁵⁰.

Out-of-field teachers are not a homogenous group. Some come to teaching mathematics willingly; others do not. Some are highly experienced practitioners in other teaching areas; others are new or recent graduates. Some see teaching mathematics as a long-term feature in their careers; others see it as short-term.

Across the country there have been a range of retraining programs designed to provide the mix of content and content pedagogical knowledge that out-of-field teachers have not been exposed to in their formal education as teachers. This is clear evidence that the education jurisdictions and others have seen targeted professional learning as the way to help improve the quality of teaching by out-of-field teachers of mathematics.

Colleen Vale and colleagues from Deakin University are at the vanguard of Australian research into out-of-field teaching of mathematics⁵¹. Their findings indicate that the best starting point is a culture in the school that recognises out-of-field teachers as important in the overall provision and health of mathematics teaching and learning in the school, and creates structures for supporting them to learn and develop. The establishment and maintenance of this culture of schools as professional learning communities requires leadership and input from the principal, mathematics leaders (inside and outside the school) and the school's other teachers of mathematics. Whilst it is realistic to acknowledge shortcomings in content and content pedagogical knowledge of out-of-field teachers of mathematics, it is important also to value the general teaching skills and pedagogies developed in other disciplines they bring to their work. These can enrich their and others' teaching of mathematics.

The findings of Vale and colleagues are that this culture is difficult to establish and maintain in schools. Time for experienced in-field teachers to mentor and otherwise support their colleagues is cited as the limiting factor. Given that improving the teaching skills of out-of-field teachers is mostly a matter of providing professional support, the question is therefore to what extent and how resources for teaching junior secondary mathematics can contribute.

⁴⁹ McKenzie, P., Rowley, G., Weldon, P. & Murphy, M. (2011). *Staff in Australia's schools 2010*. Camberwell, Vic.: ACER.

⁵⁰ The overall low rates of enrolments in higher level mathematics raises the question of how well the 'in-field' teachers as a whole are performing in this regard.

⁵¹ Personal conversation between author and Colleen Vale.

Just as their in-field colleagues, out-of-field teachers should use resources that address the broad scope of mathematics rather than a narrow focus on content skills and procedures. Resources used should incorporate contexts, promote discussion, include problem solving where more than one answer or method is possible, and promote other approaches seen to engage students in this stage of schooling.

The special feature of resources that can be valuable for out-of-field teachers is explicit annotation of materials to provide background on the content and pedagogy incorporated. These annotations, which could be text, audio or video, serve to explain *why* the resource is as it is, and prompt users to reflect on and engage with this rationale, whether as part of a professional learning community in a supportive school environment or otherwise.

This approach was pioneered in Australia by the *Mathematics Curriculum and Teaching Programme* (MCTP; 1986-90) that resulted in the development of several highly acclaimed activity banks and other resources. The MCTP resources⁵² were designed to challenge and support *all* teachers' development of their pedagogical practice. This approach has been continued to some extent in the respected Maths300 initiative.

Annotating high quality resources that have shown themselves to be effective in the classroom can make clear to users their content and pedagogical rationales. These annotations would be a substantial support for out-of-field teachers and their mentors in schools. Online publishing of resources with interactive facility would enable this information to be available as needed, with the possibility of linking to other existing professional resources to deepen out-of-field teachers' engagement with learning about teaching.

This value-adding material will also be useful to in-field teachers of mathematics as refreshers. Annotations could introduce them to orientations in content and pedagogy with which they may not be familiar.

Whilst all primary teachers will have had some pre-service education in mathematics and its teaching, many express concerns about their capacity as teachers of mathematics. In a recent survey, mathematics teacher educators in our universities shared that concern, identifying as many as a third of them as having "significant gaps" in their mathematics content knowledge as graduates⁵³. Hence, there are significant similarities between out-of-field secondary teachers of mathematics and many primary teachers. While the contexts are different, this model for resources would be equally applicable in the primary sector.

⁵² Lovitt, C. & Clarke, D. (2011) The features of a rich and balanced mathematics lesson: Teacher as designer. *Educational Designer*, 1(4). Downloaded 10 March 2015 from <http://www.educationaldesigner.org/ed/volume1/issue4/article15/index.htm>

⁵³ Morony, W. (2014). *Report on Mathematics Teacher Educators Survey June 2014*. Unpublished report to the Office of the Chief Scientist.

Question 10

How well do existing resources focus on cognitive skills in addition to mathematical content knowledge and skills – enabling students to deal with complex situations, explore, make and test conjectures, reason logically, and use a variety of mathematical methods to solve problems?

These cognitive skills are broadly encompassed in the Reasoning, Problem solving and Understanding Proficiencies of the ACM. As has been established previously, there is a vast array of resources for school mathematics. Some of these resources have a strong and clear focus on these skills, some others pay lip service only, while still more cannot even claim to go beyond content knowledge and skills.

The former category includes resources that are characterised by including open-ended or ‘rich’ tasks in mathematics. Some of the resources mentioned previously have this orientation (MCTP, Maths300, NRICH). There are others that capture this orientation, including some of the learning objects available through *Scootle*, and aspects of online commercial products such as *HotMaths* (Cambridge), *Manga High* and *Matific*. Attention to the Proficiencies needs to be an important feature of teaching mathematics in line with the ACM and therefore the curation of resources, signalled earlier as a significant need, will have these sorts of skills as one of the criteria.

The work of Sullivan and Lilburn⁵⁴ exemplifies the potential of open-ended tasks as means for teaching these skills. The author’s example below illustrates their approach:

Textbook question:

What is the overall score when the judges at a diving competition gave these scores: 3, 4, 4, 6, 5, 2?

Open-ended question:

A diver’s overall score for one dive was 4. Four of the judges gave the following scores: 3, 4, 4, 6. What scores might the other two judges have given?

Both questions deal with the same mathematical content: calculating the arithmetic mean of a few pieces of data. However, in use in the classroom, the open-ended variant provides much more scope for students to explore, use a variety of mathematical methods to solve problems, make and test conjectures, reason logically, and generalise. Through having more control of their learning, the potential for student engagement and ownership is also substantially increased.

The difference between the two variants is not great, and these authors provide strategies for turning closed questions into open ones. This is an example of a set of resources that exemplifies an approach and provides support for teachers to adopt strategies which they have seen and appreciated ‘in action’ in the classroom, that is, resources with a professional learning purpose.

However, as with all resources, it is the teacher’s use of the resource that determines whether that potential is realised. With open-ended tasks, using classroom strategies that scaffold the learning and emphasise the Proficiencies are essential to maximise student learning. This suggests that annotating open-ended resources will strengthen their impact on students’ learning by guiding teachers to use them in productive ways.

Some of the wide range of technologies available for use in the classroom can support students to ask and answer the ‘what if...?’ questions that are central to

⁵⁴ Sullivan, P. & Lilburn, P. (2004). *Open-ended maths activities: Using ‘good’ questions to enhance learning in mathematics* (2nd edition). South Melbourne, Vic.: Oxford.

developing these essential cognitive skills. In some cases, such as Wishball⁵⁵, this orientation is inherent in the resource. In others, the technology is a tool that can be applied to a task or challenge, such as using spreadsheets or other mathematically able software (for example geometrical investigations using a dynamic geometry package). These packages tend to be more applicable for use in junior secondary mathematics and above. There are some good resources that utilise the power of the technologies, but the dominance of the use of textbooks in secondary schools militates against use of technology in these powerful ways, as the textbook genre requires student experiences and pathways to be constrained.

Flewelling and Higginson (2005)⁵⁶ developed a handbook containing a detailed rationale and framework for designing and using 'rich tasks' in mathematics. Their work further elaborates the approach of Sullivan and Lilburn. The handbook, and the subsequent collections of classroom activities, provides practical support for rich tasks and open-ended questions as vehicles for teaching and learning mathematics by inquiry.

⁵⁵ <http://www.scootle.edu.au/ec/viewing/L867/index.html>

⁵⁶ Flewelling, G. & Higginson, W. (2005). *Teaching with rich learning tasks: A handbook* (2nd edition). Adelaide, SA: AAMT.

Question 11

To what extent do resources and pedagogies provide teachers with the ability to teach mathematical skills and understandings in ways that encourage transfer – embedding real-world work-related examples and technologies into lessons?

Teaching practices that encourage transfer were identified as being needed to improve work readiness of young people entering the workforce in the recent *Quantitative Skills in 21st Century Workplaces* project⁵⁷. The analysis of resources (pp. 53–57, and Attachments 1–3, pp. 60–71) describes a range of available resources in three broad categories:

- resources that provide specific examples of the use of mathematics within a workplace context;
- resources to help teachers’ understanding of the use of mathematical skills in the workplace; and
- resources that provide learning opportunities and contextualised experiences.

Most of the resources in the first two categories are from industry and training sources; the third category includes some resources from more traditional school education sources. All three categories have resources with examples that address embedding real-world, work-related situations into lessons in school mathematics. However, the project also identified significant and sophisticated use of technologies, notably spreadsheets, in a range of types of workplaces. This was surprising to several of the teacher-researchers involved, reflecting a lack of focus on spreadsheets as mathematical tools in their schools, at least. The resources identified in the analysis did not include a focus on transferring mathematical skills with spreadsheets into work contexts.

This study focussed on the upper end of schooling and the transition to work and work contexts. However, using contexts of all kinds in which students learn and apply their mathematics is supported as an important means for engaging students. Such an approach also expects students to develop the capacity to choose and use mathematics relevant to solving problems in the context. Resources that capture this context-rich approach therefore help develop students’ general capacity to transfer and use their mathematics effectively in contexts. Building on these general transfer skills as mathematics teaching and learning turns its attention to vocational pathways, requires acknowledgement of some of the key features of using mathematics in the workplace, such as working in a team, attention to levels of accuracy, and communication of ideas and findings to different audiences. These and other relevant aspects would need to be addressed in resources targeted at this area.

School-to-work transition in mathematics has not had systematic emphasis in schools or the profession. It is now emerging as an important STEM skills issue⁵⁸. Resources alone will not lead to marked improvements in students’ capacity to apply and transfer their mathematics. Hence, while carefully designed, targeted resources can be useful, developing teachers’ skills and orientation to take this area seriously will need to go hand in hand. The design of resources will also need to take ‘transfer’ as a big idea through which to address a range of individual Content Descriptions in the ACM.

⁵⁷ Documents available at <http://www.aamt.edu.au/Library/Projects/Workplace-maths-skills>.

⁵⁸ Australian Industry Group. (2013). *Lifting our Science, Technology, Engineering and Maths (STEM) Skills*. Melbourne, Vic.: Author.

Question 12

Is there a need for fresh approaches/resources to be developed? If so, why? If so, what should such resources cover – what do out of field teachers most need?

Some key themes for fresh approaches to developing new resources have been identified in several of the responses to previous questions.

Emphasise the Proficiencies and General Capabilities of the ACM

The aspirations for the ACM are expressed in its Shape Paper and Rationale. These capture a vision that is world-class and which has, through the adoption of the curriculum by the States and Territories, national commitment. The extent to which the vision for the ACM is realised in practice will only become evident over time.

The documentation of the ACM is a statement of the intended curriculum. The implemented curriculum is the teaching and learning that happens in classrooms and schools. In order to minimise the slippage between the intended and implemented curriculums, teachers and schools need support to teach the curriculum with integrity, in ways that realise the vision of the ACM. Crucially, this means teaching and learning programs in schools that give strong emphasis to the Proficiencies and General Capabilities in addition to the Content Descriptions.

Classroom resources that foreground attention to these components of the ACM will be important support for minimising the distance between implemented and intended curriculum.

Classroom assessment as a key area for support

The ‘assessed’ curriculum is what students have learnt and are able to do as a result of learning the implemented curriculum. There will always be a gap between the intended and assessed curriculum – measuring or identifying progress in all that is intended to be learnt is impractical. However, the aim must be to minimise the gap. Otherwise, the WYTIWYG principle outlined in question 3 will see the implemented curriculum drift away from the intended curriculum of the ACM.

Progress with all the important learnings needs therefore to be mapped to both inform teachers’ actions in an ongoing way, and for reporting to parents and others. This requires teachers to be able to use a range of assessment approaches and strategies in the classroom. Such assessment will have a significant element of teacher judgement. Assessment resources need to give confidence to teachers themselves, and others, that these judgements have sufficient reliability for their purposes, that the judgements are quality assured. High quality assessment resources will provide both guidance on approaches and strategies, as well as contribute to the necessary quality assurance of teacher judgements.

A focus on the ‘big ideas’

The nature and presentation of the Content Descriptions in the ACM can be interpreted as a collection of relatively isolated ‘boxes to be ticked’ in planning, teaching and assessment. This can lead to a compliance approach where the most important thing is for the teacher to assure themselves and others that they have covered the required curriculum.

Designing resources with a focus on the ‘big ideas’ of mathematics is a desirable alternative which is also feasible. Such materials would be much more directed at students learning ‘big ideas’ in coherent and connected ways. Coverage of the

curriculum as represented by the Content Descriptions, would be assured by backward mapping the learning to the ACM.

Annotations that support professional learning purposes

Implicit in all of the above themes is that teacher professional learning will be a significant component in the effectiveness of any resources that are developed. The resources developed can and should provide support for the professional learning. Annotations to the resources (whether text, video, audio, student work samples, etc.) can serve this professional learning purpose. These annotations to the resources will capture and make clear the developers' rationale for the design of the resource. Material to be included as annotations will necessarily be part of the development of well-designed classroom materials and not an added task. That is, the people who design and develop the resources will have articulated their rationale, will have piloted ideas in classrooms to generate work samples, and, potentially, recorded videos of teaching and learning sequences as part of their gathering of evaluation data from field trials and so on. This material will be available to be repurposed as the annotations.

As has been outlined earlier, these annotations will make the resources highly useful for all teachers – including out-of-field secondary teachers and many primary teachers – by linking classroom activity to content and pedagogical principles.

Having the annotations will allow the designers to provide their views of the best means for using the resource, and why that is the case. These annotations will also serve to inform whatever professional learning support is provided for teachers. Having common messages will help bring greater coherence to views of quality teaching in mathematics, something that is increasingly important in the context of nationally agreed professional standards for teachers.

Question 13

Where are the gaps in existing teacher and student resources available online to support the Australian Curriculum: Mathematics?

- Consider whether there are sufficient free, high-quality mathematics resources for primary school teachers

The most obvious collection of free online mathematics resources for primary mathematics is that available through *Scootle*⁵⁹. There are several thousand items, all tagged to ACM Content Descriptions. The search allows for filtering in terms of General Capabilities and Cross-curriculum Priorities, but interestingly not for Proficiencies.

A study⁶⁰ in 2011 concluded that the tagging and search functions of *Scootle* militates against effective use by all but the most dedicated of teachers. Simply put, the study found that the search often returns a large number of resources that may be poorly matched to the search criteria. As an added frustration for users, the same study noted varying educational quality of *Scootle* resources. The project proposed means for limiting the number of resources returned by a search to being only those that closely align to the criteria. It also proposed quality assurance criteria and processes which would serve to limit the resources available through *Scootle* to those that meet some agreed quality standards. Neither of these suggestions has been taken up by Education Services Australia (ESA), apparently due to lack of funding.

It would appear that *Scootle* resources are not being kept up to date, again probably due to changes in funding priorities. Two examples of free resources available from overseas are the very well regarded *Library of Virtual Manipulatives*⁶¹ from the USA and *NRICH*⁶² (UK). The former are included in *Scootle*, but those from *NRICH* appear not to be referenced.

Scootle as a whole is therefore one set of resources that requires curation to maximise the use of the best materials it contains. Attention to its tagging and search functions is also needed to maximise its usability. Further, if *Scootle* is to be a useful continuing repository it needs to be able to renew its collection by sourcing appropriate resources.

Resources relating to the four themes outlined above are missing from the online resources for primary mathematics. In particular, materials with a ‘big picture’ interpretation of the ACM are needed to underpin students’ successful transition to secondary school mathematics, with the major gap being in the development of multiplicative reasoning, as exemplified by the work of Siemon and colleagues⁶³.

- Consider whether there are sufficient engaging mathematics resources for junior secondary students

The discussion above regarding *Scootle* is also relevant to this level of schooling as its resources cover F–12.

Whilst not free – and not limited to junior secondary – the *Maths300* resources are well regarded, and considered as being designed with student engagement as a high priority. The *Maths300* collection is by no means comprehensive in its coverage of the

⁵⁹ <http://www.scootle.edu.au/ec/p/home>

⁶⁰ Unpublished project report to ESA on the *Australian Curriculum Connect* project conducted by AAMT in 2011.

⁶¹ <http://nlvm.usu.edu/en/nav/vlibrary.html>

⁶² <http://nrich.maths.org/frontpage>

⁶³ <http://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/assessment/Pages/scaffoldnum.aspx>

(primary and) junior secondary curriculum. The *Maths300* collection could be expanded.

The Internet contains many resources of highly variable quality and applicability in Australian schools. Again, determined curation of these against quality criteria is necessary to enable Australian teachers to take advantage of their availability. Curation of Internet resources would be an ongoing task, both to find and evaluate additions as well as monitoring the continuing availability of existing resources.

Junior secondary mathematics has been identified as an area of need for teachers and schools. There are gaps related to the themes outlined in Question 12. Filling these gaps will need to reflect pedagogies that serve to improve engagement of adolescents in mathematics, as identified earlier.

- Consider whether existing curriculum resources utilise emerging digital technologies to illustrate mathematical concepts e.g., graphical or numerical programmes that can assimilate large amounts of data to illustrate changes to inputs or outputs

There is different use of digital technologies in learning and teaching programs according to the different orientations of teachers:

- some teachers are enthusiastic early adopters whose practice can be at the leading edge internationally – they are knowledgeable about the benefits of using technology in their teaching of mathematics and committed to doing so;
- for some others, the use of technologies in high stakes assessment at year 12 and/or pressures at the school level are translated into a competent use of technologies as required; and
- many others avoid use of technologies in their teaching of mathematics when possible.

The main requirement for moving teachers from the second two orientations to being more confident, willing and competent in using technologies in their teaching of mathematics is sufficient, targeted professional learning. As argued earlier in response to Question 12, resources that promote and exemplify effective use of technologies for teaching and learning mathematics, and that are annotated to make pedagogical principles and processes clear, would be valuable within any professional learning programs for these teachers. These resources could be designed to use open-source (i.e. free of charge) dynamic mathematics software packages such as Geogebra⁶⁴, which has many users and devotees among teachers in Australia.

- Are there other gaps in resources for teachers and/or students?

Assessment has been identified as a key area for provision of resources in the response to Question 12. Resources for classroom assessment for use by teachers as part of their work are the key need, with support for teacher judgements about students' learning an important feature. Of particular importance for mathematics in the context of the ACM, is support for teachers to assess all the Proficiencies and the relevant General Capabilities, thereby signalling the importance of these to success and progress in mathematics.

The *Supporting the Australian Curriculum Online* (SACOL) initiative was conducted by Education Services Australia in the period 2012-13. SACOL supported a project by AAMT entitled *Top Drawer Teachers*. The genesis of that project was

⁶⁴ <http://www.geogebra.org>

consultation to identify mathematics topics that are difficult to teach. From the list of 20 or so, the funding enabled the development of professional resources for teachers in six areas (Mental computation, Geometric reasoning, Patterns, Fractions, Statistics and Reasoning). Each collection of resources links existing or purpose-developed classroom resources to give users the means to address the key challenges common to teaching any area of mathematics – identifying and focusing on the ‘big ideas’; understanding and dealing with common student misconceptions; good teaching; and assessment. The resources are linked to the relevant research for those teachers who wish to further their own knowledge through professional reading.

The *Top Drawer Teacher* resources were developed within the parameters of the SACOL project and the model may need modification in a future program of resource development. However, approaching the task of developing resources for teachers that address the known ‘difficult to teach’ areas would certainly be ‘filling gaps’. These include negative numbers; arithmetic foundations for algebra; introducing formal algebra; trigonometry; language and communication in mathematics; and three-dimensional geometry and visualisation.

- Consider whether the gaps could be filled by existing resources that are not formally linked to the Australian Curriculum (in use in Australia or overseas) (including consideration of whether such resources are free for education, evidence of teacher satisfaction and evidence of improved student learning outcomes)

From Australia

The NSW and Victoria governments have extensive resources on their websites. Those jurisdictions have chosen to implement the ACM through localised versions (a syllabus in NSW, and AusVELS, where VELS stands for the Victorian Essential Learning Standards in that state) to which these resources are linked. It would be desirable for these resources to be available nationally, with links to the ACM.

A number of the resources referred to in Question 11 in relation to work-related examples were developed by industry and training and do not generally link to the ACM, yet these are valued by some teachers and used effectively in some settings. Identifying these links is quite feasible.

Many of the original MCTP activities are relevant in 2015 as examples of good practice and could be revised and updated⁶⁵.

From overseas

The NZ Numeracy site (Ministry of Education) contains extensive support materials that are already in use in this country. Many of the resources are highly applicable in the context of the ACM and Australian teaching and learning of mathematics.

- Consider which age groups could most benefit from the development of new types of resources and the rationale/evidence for such approaches

This matter has been addressed in the response to Question 12 and in other places in this paper. To reiterate, key emphases need to be the ‘big ideas’, the Proficiencies and General Capabilities in mathematics, and exemplifying and supporting improved classroom assessment. Whilst there are needs at all levels of schooling, junior secondary mathematics appears to have the greatest and most pressing needs.

⁶⁵ It is likely that the Commonwealth of Australia owns the copyright on the materials.

Resources need to be carefully designed to support professional learning purposes through annotations that highlight their features.

Question 14

Are there any other issues of critical importance that you are aware of?

There are at least three areas that may be worthy of further consideration.

Parental engagement and support

Two distinct but related areas in which parents and caregivers can be involved in mathematics are:

- parents engaging with mathematics through every day activities in the home to support their children's learning;
- enlisting parents in the efforts to address the societal factors that discourage young people, and especially females, taking STEM pathways, and the need for higher level mathematics for these.

Maths anxiety

This is an established issue for many, particularly girls, that militates against feeling positive about their mathematics and intentions for further study in the subject. Some of the issues were recently canvassed by Wilson in an article in *The Conversation*⁶⁶.

Students with disabilities

This would appear to have been a significantly neglected area in mathematics education research in Australia. It is not clear whether there is enough research of substance that can be translated into resources. There is no clear evidence that this matter has been a particular focus overseas, but further investigation may be warranted.

⁶⁶ Wilson, S. (2015). Teachers' gender bias in maths affects girls later. Downloaded from <http://theconversation.com/teachers-gender-bias-in-maths-affects-girls-later-37844>.